A study on the Nonlinear Normal Mode Vibration Using Adelphic Integral

Huinam Rhee*
School of Mechanical and Automotive Engineering, Sunchon National University, 315 Maegok-dong Sunchon, Junnam 540-742, Korea
Jeong-Soo Kim
Satellite R&D Division Korea Aerospace Research Institute, 45 Eoeun-Dong, Yuseong-Gu, Daejeon 305-333, Korea

Nonlinear normal mode (NNM) vibration, in a nonlinear dual mass Hamiltonian system, which has 6^{th} order homogeneous polynomial as a nonlinear term, is studied in this paper. The existence, bifurcation, and the orbital stability of periodic motions are to be studied in the phase space. In order to find the analytic expression of the invariant curves in the Poincare Map, which is a mapping of a phase trajectory onto 2 dimensional surface in 4 dimensional phase space, Whittaker's Adelphic Integral, instead of the direct integration of the equations of motion or the Birkhoff-Gustavson (B-G) canonical transformation, is derived for small value of energy. It is revealed that the integral of motion by Adelphic Integral is essentially consistent with the one obtained from the B-G transformation method. The resulting expression of the invariant curves can be used for analyzing the behavior of NNM vibration in the Poincare Map.

Key Words : Adelphic Integral, Nonlinear Normal Mode Vibration, Poincare Map, Hamiltonian, Action-Angle Variable, Birkhoff-Gustavson Canonical Transformation, Bifurcation, Internal Resonance

1. Introduction

The existence, bifurcation, and the orbital stability of periodic motions, which is called nonlinear normal mode, in a nonlinear dual mass Hamiltonian system, which has 6^{th} order homogeneous polynomial as a nonlinear term as shown in Fig. 1, are under consideration in this paper. In the previous work (Rhee, 1999) the dynamical structure of the same oscillator, was investigated by picturing the Poincare Map, which is a mapping of a phase trajectory onto 2 dimensional surface in 4 dimensional phase space, by direct integration of the equations of motion, and also by generating an approximation for the Poincare Map via Birkhoff-Gustavson (Birkhoff, 1927; Gustavson, 1963 ; Month et. al., 1980) canonical transformation (Arnold, 1978) for small values of energy. In that work, particularly, the existence and the stability of Nonlinear Normal Mode was studied, and it was found that the system has 2 or 4 Similar Nonlinear Normal Modes depending on the values of the nonlinear parameter k. The bifurcating modes enter as stable while the mode from which they bifurcated changes from condition to unstable condition.

In this paper, in order to find the analytic expression of the invariant curves in the Poincare Map, Whittaker's Adelphic Integral (Whittaker, 1989), instead of the direct integration of the equations of motion, or the Birkhoff-Gustavson canonical transformation, is derived for small value of energy. We will show that although the

* Corresponding Author,
E-mail : hnrhee@sunchon.ac.kr
TEL : +82-61-750-3824; **FAX** : +82-61-750-3820
School of Mechanical and Automotive Engineering, Sunchon National University, 315 Maegok-dong Sunchon, Junnam 540-742, Korea. (Manuscript **Received** February 18, 2003; **Revised** September 19, 2003)

calculation process is so much complicated, the resulting integral of motion obtained by Adelphic Integral is consistent with the one obtained from the B-G transformation method.

2. Whittaker's Adephic Integral and Relationship with Birkhoff-Gustavson's Integral

The method using Adelphic Integral, was developed by Whittaker (Whittaker, 1989), and starts with a canonical transformation which is defined in terms of action-angle variables. This method is based on the fact that the Poisson bracket of any integral ϕ and the Hamiltonian H equals zero. The resulting integral ϕ is known as Adelphic integral. Whittaker has divided the method into different cases. The cases arise from problems associated with internal resonance. In this method zero divisors are associated with internal resonances. Because the system shown in Fig. 1 has a 1 : 1 internal resonance, we will consider only internal resonance case in this paper.

Let us begin with a generalized form of Hamiltonian as follows :

$$H(u, v)=H(2)(u, v)+H(3)(u, v)+H(4)(u, v)+\cdots \quad (1)$$

where the quadratic term is of the form $H(2)(u, v)=\sum_{v=1}^{n}(\alpha_v^2 u_v^2+v_v^2)/2$, and $H(3)$ and $H(4)$ are cubic and quartic polynomials of u and v, respectively.

We can easily transform from (u, v) to (x, y) used in the previous work (Rhee, 1999), with the following canonical transformation ;

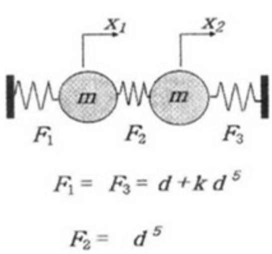


Fig. 1 Nonlinear oscillator which has a 5th order nonlinearity in stiffenss

$$u_v=x_v/\alpha^{1/2}, \quad v_v=\alpha^{1/2}y_v \quad (2)$$

First, we define canonical transformation from (u, v) to (Q, P);

$$u=(2Q)^{1/2}\alpha^{-1/2}\cos P \quad (3a)$$

$$v=(2\alpha Q)^{1/2}\sin P \quad (3b)$$

The variables Q and P are known as action-angle variables (Arnold, 1979 ; Whittaker, 1989 ; Goldstein, 1950). From the inverse transformation of Eq. (3), we see that the action variables Q corresponds to amplitude and the angle variable P corresponds to the polar angle locating the trajectory in the (u, v) phase space.

Let us assume the system has two degree of freedom. In terms of the new variables (Q, P), $H(2)(u, v)$ becomes

$$\tilde{H}(2)(Q, P)=(\alpha_1 Q_1+\alpha_2 Q_2) \quad (4)$$

and $H(s)(u, v)$ becomes a sum of terms proceeding in powers of $Q_1^{1/2}$ and $Q_2^{1/2}$ and in trigonometric functions of multiples of P_1 and P_2 ; that is, terms of the type

$$Q_1^{m/2}Q_2^{n/2}\cos(ip_1+ip_2), \ s=m+n \quad (5)$$

where m and n are nonnegative integers, and $m-|i|$, $n-|j|$ are zero or an even integer.

We call $s=m+n$ the order of the term. The general form of $\tilde{H}(6)(Q, P)$ is as follows :

$$\begin{aligned}
&Q_1^3(Y_1+Y_2\cos 2P_1+Y_3\cos 4P_1+Y_4\cos 6P_1)\\
&+Q_1^{\frac{5}{2}}Q_2^{\frac{1}{2}}\{Y_5\cos(P_1+P_2)+Y_6\cos(P_1-P_2)\\
&\quad+Y_7\cos(3P_1+P_2)+Y_8\cos(3P_1-P_2)\\
&\quad+Y_9\cos(5P_1+P_2)+Y_{10}\cos(5P_1-P_2)\}\\
&+Q_1^2Q_2\{Y_{11}+Y_{12}\cos 2P_1+Y_{13}\cos 2P_2\\
&\quad+Y_{14}\cos(2P_1+2P_2)+Y_{15}\cos(2P_1-2P_2)\\
&\quad+Y_{16}\cos 4P_1+Y_{17}\cos(4P_1+2P_2)\\
&\quad+Y_{18}\cos(4P_1-2P_2)\}\\
&+Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\{Y_{19}\cos(P_1+P_2)+Y_{20}\cos(P_1-P_2)\\
&\quad+Y_{21}\cos(3P_1+P_2)+Y_{22}\cos(3P_1-P_2)\\
&\quad+Y_{23}\cos(P_1+3P_2)+Y_{24}\cos(P_1-3P_2)\\
&\quad+Y_{25}\cos(3P_1+3P_2)+Y_{26}\cos(3P_1-3P_2)\}\\
&+Q_1Q_2^2\{Y_{27}+Y_{28}\cos 2P_1+Y_{29}\cos 2P_2\\
&\quad+Y_{30}\cos(2P_1+2P_2)+Y_{31}\cos(2P_1-2P_2)\\
&\quad+Y_{32}\cos 4P_2+Y_{33}\cos(2P_1+4P_2)\\
&\quad+Y_{34}\cos(2P_1-4P_2)\}
\end{aligned} \quad (6)$$

$$+Q_1^{\frac{1}{2}}Q_2^{\frac{5}{2}}\{\,Y_{35}\cos(P_1+P_2)+Y_{36}\cos(P_1-P_2)$$
$$+Y_{37}\cos(P_1+3P_2)+Y_{38}\cos(P_1-3P_2)$$
$$+Y_{39}\cos(P_1+5P_2)+Y_{40}\cos(P_1-5P_2)\}$$
$$+Q_2^3(\,Y_{41}+Y_{42}\cos 2P_2+Y_{43}\cos 4P_2+Y_{44}\cos 6P_2)$$

where Y_i's are coefficients.

If $\phi(Q, P)$ =const. is an integral we must have

$$\begin{aligned}\frac{d\phi}{dt}&=\frac{\partial\phi}{\partial Q}\dot{Q}+\frac{\partial\phi}{\partial P}\dot{P}\\&=\frac{\partial\phi}{\partial Q}\frac{\partial H}{\partial P}-\frac{\partial\phi}{\partial P}\frac{\partial H}{\partial Q}=(\phi,\,H)=0\end{aligned}\tag{7}$$

where tildes are omitted for convenience.

The notation in Eq. (7) is referred as the *Poisson bracket*. We expand Eq. (7) and equate terms of equal order. For example, if $H=H(2)+H(6)$ then from the Eq. (7) we have

$$\begin{aligned}&(\phi(2)+\phi(4)+\phi(6)+\phi(8)+\phi(10)+\cdots,\\&H(2)+H(6))=0\end{aligned}\tag{8}$$

Equating terms of equal order, we have

$$(\phi(2),\,H(2))=0\tag{9a}$$

$$(\phi(4),\,H(2))=0\tag{9b}$$

$$(\phi(6),\,H(2))=-(\phi(2),\,H(6))\tag{9c}$$

$$(\phi(8),\,H(2))=-(\phi(4),\,H(6))\tag{9d}$$

$$(\phi(10),\,H(2))=-(\phi(6),\,H(6))\tag{9e}$$

It is noted from Eq. (9a) that

$$\alpha_1\frac{\partial\phi(2)}{\partial P_1}+\alpha_2\frac{\partial\phi(2)}{\partial P_2}=0\tag{10}$$

Let us assume

$$\phi(2)=\alpha_1Q_1-\alpha_2Q_2\tag{11}$$

which certainly satisfies Eq. (10), and

$$\phi(4)=\phi(8)=0$$

which satisfies Eqs. (9b) and (9c).

Substituting Eq. (11) into Eq. (9c), we have

$$\alpha_1\frac{\partial\phi(6)}{\partial P_1}+\alpha_2\frac{\partial\phi(6)}{\partial P_2}=\alpha_1\frac{\partial H(6)}{\partial P_1}-\alpha_2\frac{\partial H(6)}{\partial P_2}\tag{12}$$

This implies that to any term $A\cos(iP_1+jP_2)$ in $H(6)$ there corresponds a term $\{(i\alpha_1-j\alpha_2/i\alpha_1+j\alpha_2)\}A\cos(iP_1+jP_2)$ in $\phi(6)$.

Therefore, we can state the general form of ϕ as follows:

$$\begin{aligned}\phi&=\phi(2)+\phi(6)+\cdots\\
&=\alpha_1Q_1-\alpha_2Q_2+Q_1^3(\,Y_2\cos 2P_1+Y_3\cos 4P_1+Y_4\cos 6P_1)\\
&\quad+Q_1^{\frac{5}{2}}Q_2^{\frac{1}{2}}\Big\{\frac{(\alpha_1-\alpha_2)}{(\alpha_1+\alpha_2)}Y_5\cos(P_1+P_2)+\frac{(\alpha_1+\alpha_2)}{(\alpha_1-\alpha_2)}Y_6\cos(P_1-P_2)\\
&\qquad+\frac{(3\alpha_1-\alpha_2)}{(3\alpha_1+\alpha_2)}Y_7\cos(3P_1+P_2)+\frac{(3\alpha_1+\alpha_2)}{(3\alpha_1-\alpha_2)}Y_8\cos(3P_1-P_2)\\
&\qquad+\frac{(5\alpha_1-\alpha_2)}{(5\alpha_1+\alpha_2)}Y_9\cos(5P_1+P_2)+\frac{(5\alpha_1+\alpha_2)}{(5\alpha_1-\alpha_2)}Y_{10}\cos(5P_1-P_2)\Big\}\\
&\quad+Q_1^2Q_2\Big\{\,Y_{12}\cos 2P_1+Y_{13}\cos 2P_2+\frac{(2\alpha_1-2\alpha_2)}{(2\alpha_1+2\alpha_2)}Y_{14}\cos(2P_1+2P_2)\\
&\qquad+\frac{(2\alpha_1+2\alpha_2)}{(2\alpha_1-2\alpha_2)}Y_{15}\cos(2P_1-2P_2)+Y_{16}\cos 4P_1\\
&\qquad+\frac{(4\alpha_1-2\alpha_2)}{(4\alpha_1+2\alpha_2)}Y_{17}\cos(4P_1+2P_2)+\frac{(4\alpha_1+2\alpha_2)}{(4\alpha_1-2\alpha_2)}Y_{18}\cos(4P_1-2P_2)\Big\}\\
&\quad+Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\Big\{\frac{(\alpha_1-\alpha_2)}{(\alpha_1+\alpha_2)}Y_{19}\cos(P_1+P_2)+\frac{(\alpha_1+\alpha_2)}{(\alpha_1-\alpha_2)}Y_{20}\cos(P_1-P_2)\\
&\qquad+\frac{(3\alpha_1-\alpha_2)}{(3\alpha_1+\alpha_2)}Y_{21}\cos(3P_1+P_2)+\frac{(3\alpha_1+\alpha_2)}{(3\alpha_1-\alpha_2)}Y_{22}\cos(3P_1-P_2)\\
&\qquad+\frac{(\alpha_1-3\alpha_2)}{(\alpha_1+3\alpha_2)}Y_{23}\cos(P_1+3P_2)+\frac{(\alpha_1+3\alpha_2)}{(\alpha_1-3\alpha_2)}Y_{24}\cos(P_1-3P_2)\\
&\qquad+\frac{(3\alpha_1-3\alpha_2)}{(3\alpha_1+3\alpha_2)}Y_{25}\cos(3P_1+3P_2)+\frac{(3\alpha_1+3\alpha_2)}{(3\alpha_1-3\alpha_2)}Y_{26}\cos(3P_1-3P_2)\Big\}\\
&\quad+Q_1Q_2^2\Big\{\,Y_{28}\cos 2P_1-Y_{29}\cos 2P_2+\frac{(2\alpha_1-2\alpha_2)}{(2\alpha_1+2\alpha_2)}Y_{30}\cos(2P_1+2P_2)\\
&\qquad+\frac{(2\alpha_1+2\alpha_2)}{(2\alpha_1-2\alpha_2)}Y_{31}\cos(2P_1-2P_2)-Y_{32}\cos 4P_2\\
&\qquad+\frac{(2\alpha_1-4\alpha_2)}{(2\alpha_1+4\alpha_2)}Y_{33}\cos(2P_1+4P_2)+\frac{(2\alpha_1+4\alpha_2)}{(2\alpha_1-4\alpha_2)}Y_{34}\cos(2P_1-4P_2)\\
&\quad+Q_1^{\frac{1}{2}}Q_2^{\frac{5}{2}}\Big\{\frac{(\alpha_1-\alpha_2)}{(\alpha_1+\alpha_2)}Y_{35}\cos(P_1+P_2)+\frac{(\alpha_1+\alpha_2)}{(\alpha_1-\alpha_2)}Y_{36}\cos(P_1-P_2)\\
&\qquad+\frac{(\alpha_1-3\alpha_2)}{(\alpha_1+3\alpha_2)}Y_{37}\cos(P_1+3P_2)+\frac{(\alpha_1+3\alpha_2)}{(\alpha_1-3\alpha_2)}Y_{38}\cos(P_1-3P_2)\\
&\qquad+\frac{(\alpha_1-5\alpha_2)}{(\alpha_1+5\alpha_2)}Y_{39}\cos(P_1+5P_2)+\frac{(\alpha_1+5\alpha_2)}{(\alpha_1-5\alpha_2)}Y_{40}\cos(P_1-5P_2)\Big\}\\
&\quad+Q_2^3(-Y_{42}\cos 2P_2-Y_{43}\cos 4P_2-Y_{44}\cos 6P_2)+\cdots\end{aligned}\tag{13}$$

It should be noted that the Adelphic integral in Eq. (13) is well defined if we have no internal resonance. However, the system considered in this paper has internal resonance ($\alpha_1=\alpha_2=1$), so we have zero divisors in the expression of ϕ. That is, we obtain $\phi=\phi(2)+\dfrac{\phi(6)}{D}+\cdots$, but $\phi(6)$ has terms with vanishing denominators, $D=0$. The first integral in this case becomes $\phi=\phi(6)+\phi(10)+\cdots$, where $\phi(6)$ consists of those terms that contributed to $D=0$, neglecting arbitrary constants. Therefore,

$$\begin{aligned}\phi(6) &= Q_1^{\frac{5}{2}}Q_2^{\frac{1}{2}}Y_6\cos(P_1-P_2)+Q_1^2Q_2Y_{15}\cos(2P_1-2P_2)\\ &+Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\{Y_{20}\cos(P_1+P_2)+Y_{26}\cos(3P_1-3P_2)\}\\ &+Q_1Q_2^2Y_{31}\cos(2P_1-2P_2)+Q_1^{\frac{1}{2}}Q_2^{\frac{5}{2}}Y_{36}\cos(P_1-P_2)\end{aligned} \tag{14}$$

If $f(Q,\ P)+\dfrac{g(Q,\ P)}{\mu}=const.=\gamma$ is a first integral, then multiplying by μ and taking the limit, as $\mu\rightarrow 0$, $\gamma\rightarrow\infty$ we can obtain $g(Q,\ P)=\lim_{\mu\rightarrow 0}(\mu\gamma)=const$ as the desired form of the integral when $\mu=0$.

We now add the following terms

$$C_1Q_1^3+C_2Q_1^2Q_2+C_3Q_1Q_2^2+C_4Q_2^3$$

which is the complementary solution of Eq. (12), to $\phi(6)$ and determine the arbitrary constants C_1, C_2, C_3 and C_4 by requiring that terms with vanishing denominators disappear from higher order term of ϕ.

Therefore, $\phi(6)$ becomes as follows :

$$\begin{aligned}\phi(6) &= Q_1^{\frac{5}{2}}Q_2^{\frac{1}{2}}Y_6\cos(P_1-P_2)+Q_1^2Q_2Y_{15}\cos(2P_1-2P_2)\\ &+Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\{Y_{20}\cos(P_1-P_2)+Y_{26}\cos(3P_1-3P_2)\}\\ &+Q_1Q_2^2Y_{31}\cos(2P_1-2P_2)+Q_1^{\frac{1}{2}}Q_2^{\frac{5}{2}}Y_{36}\cos(P_1-P_2)\\ &+C_1Q_1^3+C_2Q_1^2Q_2+C_3Q_1Q_2^2+C_4Q_2^3\end{aligned} \tag{15}$$

To solve for the constants C_1, C_2, C_3 and C_4, it is required that by Eq. (9e)

$$(\phi(10),\ H(2))=-(Q(6),\ H(6))$$

where $\phi(6)$ is given by Eq. (15).

First, we expand the right hand side of Eq. (9e)

$$\begin{aligned}&\frac{\partial\phi(6)}{\partial Q_1}\frac{\partial H(6)}{\partial P_1}-\frac{\partial\phi(6)}{\partial P_1}\frac{\partial H(6)}{\partial Q_1}\\ &+\frac{\partial\phi(6)}{\partial Q_2}\frac{\partial H(6)}{\partial P_2}-\frac{\partial\phi(6)}{\partial P_2}\frac{\partial H(6)}{\partial Q_2}\end{aligned} \tag{16}$$

and find the coefficients of the terms that contribute to $\sin(P_1-P_2)$, $\sin(2P_1-2P_2)$, $\sin(3P_1-3P_2)$, $\cdots$, and then require them vanish.

Finally, we find

$$\begin{aligned}&C_1=C_4=C.\\ &C_2=C_3=3C-(5/4)(k+1)+(15/4)\end{aligned} \tag{17}$$

Thus, Whittaker's Adelphic integral for our nonlinear oscillator is

$$\begin{aligned}\phi &= \phi(6)\\ &=(-5/2)Q_1^{\frac{5}{2}}Q_2^{\frac{1}{2}}\cos(P_1-P_2)+(5/2)Q_1^2Q_2\cos(2P_1-2P_2)\\ &+(-15/2)Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\cos(P_1-P_2)-(5/6)Q_1^{\frac{3}{2}}Q_2^{\frac{3}{2}}\cos(3P_1-3P_2)\}\\ &+(5/2)Q_1Q_2^2\cos(2P_1-2P_2)-(5/2)Q_1^{\frac{1}{2}}Q_2^{\frac{5}{2}}\cos(P_1-P_2)\\ &+C_1Q_1^3+C_2Q_1^2Q_2C_3Q_1Q_2^2+C_4Q_2^3\end{aligned} \tag{18}$$

where C_1, C_2, C_3 and C_4 are related by Eq. (17) and the constants Y_6, Y_{15}, Y_{20}, Y_{31}, and Y_{36} are obtained by the Hamiltonian

$$\begin{aligned}H=&(1/2)(u_1^2+v_1^2+u_2^2+v_2^2)+(k/6)(u_1^6+u_2^6)\\ &+(1/6)(u_1-u_2)^6\end{aligned} \tag{19}$$

In order to write ϕ in terms of the original coordinates $(u,\ v)$, we use the inverse transformation

$$\tan P=v/u$$

$$Q=(u^2+v^2)/2$$

Thus we obtain

$$\begin{aligned}\phi=&(-5/16)\{(u_1^2+v_1^2)^2+3(u_1^2+v_1^2)(u_2^2+v_2^2)+(u_2^2+v_2^2)^2\}\\ &\times(u_1u_2+v_1v_2)\\ &+(5/16)(u_1^2+v_1^2+u_2^2+v_2^2)+(u_1^2-v_1^2)(u_2^2-v_2^2)\\ &+(5/4)(u_1^2+v_1^2+u_2^2+v_2^2)u_1u_2v_1v_2\\ &+(-5/48)(u_1^3-3u_1v_2^2)(u_2^3-3u_2v_2^2)+(3u_1^2v_1-v_1^3)(3u_2^2v_2-v_2^3)\}\\ &+(C/8)\{(u_1^2+v_1^2)^3+(u_2^2+v_2^2)^3\}\\ &+(1/32)(12C-5k+10)(u_1^2+v_1^2)(u_2^2+v_2^2)(u_1^2+v_1^2+u_2^2+v_2^2)\end{aligned} \tag{20}$$

It is noted that if we compare the above equation to Eq. (29) in the reference (Rhee, 1999), we see that if $C=(5/12)(k+1)$ then Whittaker's Adelphic integral (ϕ_w) and Birkhoff-Gustavson's (ϕ_{B-G}) integrals are identical. Therefore, if we take $C=\lambda+(5/12)(k+1)$ so that $\lambda=0$ corresponds to $\phi_w=\phi_{B-G}$. Then, it can be seen that

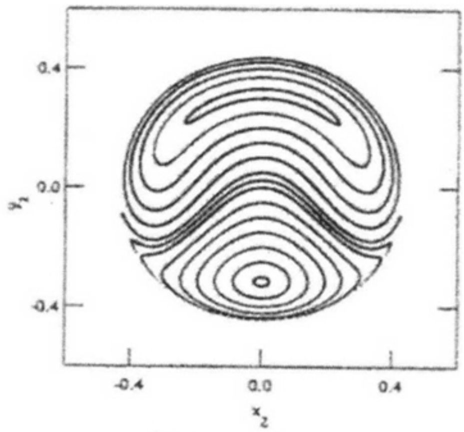
$$\phi_w=\phi_{B-G}+\frac{\lambda}{8}+[(u_1^2+v_1^2)+(u_2^2+v_2^2)]^3$$

which means that

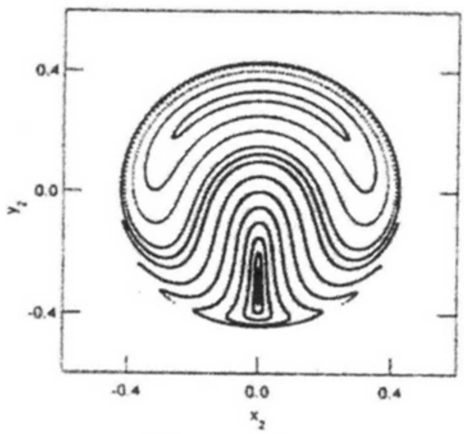
$$\phi_w=\phi_{B-G}+\lambda[H^{(2)}(u,\ v)]^3 \tag{21}$$

Thus ϕ_w and ϕ_{B-G} differ to $O(6)$ by a cubic function of the Hamiltonian.

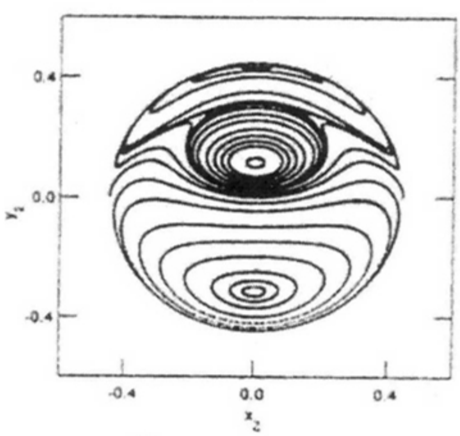
We can construct the analytic expression of the invariant curves in the Poincare Map by combining the integral of motion in Eq. (20) with the Hamiltonian in Eq. (19) as discussed in the reference (Rhee, 1999). The detailed procedure to calculate the expression is omitted in this paper. Using the resulting analytic expression we can



(a) h=0.1, k=4



(b) h=0.1, k=8



(c) h=0.1, k=−1

Fig. 2 Invariant curves in the Poincare Map calculated using the integral of motion

easily construct the Poincare Map. As can be seen in the figures in the reference (Rhee, 1999), the analytic expression represents essentially identical invariant curves compared to the Poincare Map obtained by the direct integration of the equations of motion for small value of energy. Fig. 2 shows an example of the level lines calculated by Eqs. (19) and (20). As a result we can see that the nonlinear dual mass Hamiltonian system, which has 6^{th} order homogeneous polynomial as a nonlinear term considered in this paper has 2 or 4 Similar Nonlinear Normal Modes depending on the values of the nonlinear parameter k. The bifurcating modes enter as stable while the mode from which they bifurcated changes from stable condition to unstable condition.

3. Conclusions

Nonlinear normal mode vibration, in a nonlinear dual mass Hamiltonian system, which has 6^{th} order homogeneous polynomial as a nonlinear term, is studied in this paper. In order to find the analytic expression of the invariant curves in the Poincare Map, Whittaker's Adelphic Integral, instead of the direct integration of the equations of motion or the Birkhoff-Gustavson canonical transformation, is derived for small value of energy. It is revealed that the integral of motion by Adelphic Integral is essentially consistent with the one obtained from the B-G transformation method. They differ to the order of 6 by a cubic function of the Hamiltonian. The resulting expression of the invariant curves can be used for analyzing the behavior of NNM vibration in the Poincare Map. It can be clearly seen that the system considered in this paper has 2 or 4 Similar Nonlinear Normal Modes depending on the values of the nonlinear parameter. The bifurcating modes are stable while the mode from which they bifurcated changes from stable condition to unstable condition.

Acknowledgment

This work was supported by Research Foundation of Engineering College, Sunchon National University, and by Brain Korea 21 project in 2003.

References

Arnold, V. I., 1978, "Mathematical Methods of Classical Mechanics," Springer-Verlag.

Birkhoff, G. D., 1927, "Dynamical Systems," AMS Colloquim Publication.

Goldstein, H., 1950, "Classical Mechanics," Addison-Wesley.

Gustavson, F. G., 1963, "On constructing Formal Integrals of a Hamiltonian Systems Near an Equilibrium Point," The Astronomical Journal.

Huinam Rhee, 1999, "On the Study of Nonlinear Normal Mode vibration via Poincare Map and Integral of Motion," Journal of Korean Society of Noise and Vibration Engineering, Vol. 9 (1), pp. 196~205.

Month, L. A. and Rand, R. H., 1980," Application of the Poincare Map to the stability of Nonlinear Normal Modes," Journal of Applied Mechanics.

Whittaker, E. T., 1989, "A Treatise on the Analytical Dynamics of Particles and Rigid Bodies," 4ed., Cambridge Univ Press.